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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 8: GNSS field measurement systems in real-time kinematic (RTK)

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 8: Systèmes de mesure GNSS sur site en temps réel cinématique

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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

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ISO 17123-8 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*:

— *Part 1: Theory*

— *Part 2: Levels*

— *Part 3: Theodolites*

— *Part 4: Electro-optical distance meters (EDM instruments)*

— *Part 5: Electronic tacheometers*

— *Part 6: Rotating lasers*

— *Part 7: Optical plumbing instruments*

— *Part 8: GNSS field measurement systems in real-time kinematic (RTK)*

Introduction

This part of ISO 17123 can be thought of as one of the first steps in the process of evaluating the uncertainty of measurements (more specifically of measurands). The uncertainty of a result of a measurement is dependent on a number of factors. These include among others: repeatability, reproducibility (between day repeatability) and a thorough assessment of all possible error sources, as prescribed by the ISO Guide to the expression of uncertainty in measurement (GUM).

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposely designed to minimize atmospheric influences.

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 8: GNSS field measurement systems in real-time kinematic (RTK)

1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of Global Navigation Satellite System (GNSS) field measurement systems (this includes GPS, GLONASS as well as the future systems like GALILEO) in real-time kinematic (GNSS RTK) and their ancillary equipment when used in building, surveying and industrial measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the required application at hand, and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

ISO 17123-2, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 2: Levels*

ISO 17123-5, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 5: Electronic tacheometers*

GUM, *Guide to the expression of uncertainty in measurement, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML*, 1993, corrected and reprinted in 1995

VIM, *International vocabulary of basic and general terms in metrology, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML*, 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 9849, ISO 17123-1, ISO 17123-2, ISO 17123-5, GUM and VIM apply.

4 General

4.1 Preamble

The real-time kinematic positioning method is a relative measuring procedure using reference (base) and moving (rover) receivers. For utilization of network RTK applications, a separate reference receiver is not required. Both receivers perform the observations simultaneously and merge their results by wireless transmission. Thus, the rover can display the instantaneous coordinates of the antenna in any appropriate datum e.g. ITRF (International Terrestrial Reference Frame). For practical use they are transformed to horizontal coordinates and ellipsoidal heights. Subsequently, only these types of coordinate are treated as original observables.

4.2 Requirements

Before commencing surveying, it is important for the operator to ensure that the equipment, the GNSS receiver and antenna, has sufficient precision for the task required.

The test should apply typically to a set of GNSS receivers and antennae listed in the manufacturer's reference manual. In case of using network RTK, consistency of antenna models (eg. antenna correction parameters) shall be ensured.

The receiver, antenna and their ancillary equipment for rovers points shall be checked to be in acceptable condition according to the methods specified in the reference manual.

The operator shall follow the guidelines in the manufacturer's reference manual for positioning requirements such as the minimum number of satellites, maximum PDOP (Position Dilution Of Precision) value, minimum observation time and possibly other required pre-conditions.

The operator shall initialize the receiver by resetting its power prior to every measurement and collect the data after the integer ambiguity fixing has been completed.

The following is the guideline for achievable centring precision expressed in standard deviation:

— centering: 1 mm;

— measuring the antenna height: 1 mm.

The results of the test are influenced by several factors, such as satellite configuration visible at the points, ionospheric and tropospheric conditions, multipath environment around the points, precision of the equipment, quality of the software running in the rover equipment or in the system generating the data transmitted from the base point.

This part of ISO 17123 describes two different field procedures, namely the simplified test procedure and the full test procedure, as given in Clauses 5 and 6 respectively. Therefore, the observation time of test procedure shall be so arranged to cover such variations.

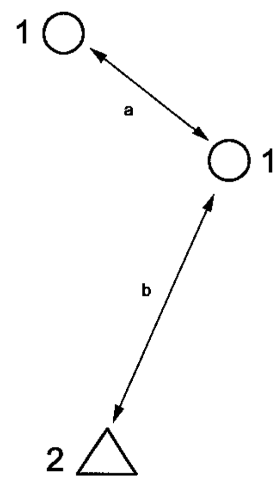
The operator shall choose the procedure that is most appropriate to the requirements of the project.

4.3 The concept of the test procedures

The test field consists of a base point and two rover points. The location of the rover points shall be close to the area of the task concerned. The separation of two rover points shall be a minimum of 2 m and shall not exceed 20 m. The positions of two rover points may be selected at convenience in the field (see Figure 1).

The horizontal distance and height difference between two rover points shall be determined by methods with precision better than 3 mm other than RTK. These values are considered as nominal values and are used in the first step of both test procedures. The horizontal distances and height differences calculated from the measured coordinates in each set of measurements shall be compared with these values in order to ensure that the measurements are free from any outlier. However, the nominal values are not used in the statistical tests.

A series of measurements consists of five sets. Each set of measurements consists of successive measurements at rover point 1 and 2.



Key

1 rover point

2 base point

a Minimum 2 m; shall not exceed 20 m.

b Corresponding distance according to the task concerned.

Figure 1 — Configuration of the field test network

The time lag between successive sets shall be approximately 5 min. This requirement makes the span of a series of measurements to be about 25 min and five sets of measurements at both rover points shall be uniformly distributed in this span. Due to the fact that the variation cycle of a typical multipath influence is about 20 min, the measuring procedure will mostly cover the period of this influence factor.

The start time for each successive series shall be separated by at least 90 min. Thus, multiple series of measurements tend to reflect influences such as changes in satellite configuration and variations in the ionospheric and tropospheric conditions.

The standard deviations calculated over all measurements will therefore represent a quantitative measure of precision in use including most of the typical influences in satellite positioning.

The simplified test procedure contains only one series of measurements and therefore only deals with the outlier detection and with no statistical evaluation. Conversely, the full test procedure consists of three series and additionally enables the estimation of standard deviations and statistical tests.

4.4 Procedure 1: Simplified test procedure

The simplified test procedure consists of a single series of measurements and provides an estimate as to whether the precision of the equipment in use is within a specified allowable deviation.

The simplified test procedure is based on a limited number of measurements. Therefore, a significant standard deviation cannot be obtained and the statistical tests are not applied. If a more precise assessment of the equipment is required, it is recommended to adopt the more rigorous full test procedure as given in 4.5.

4.5 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of the equipment in use.

The full test procedure consists of three series of measurements.

The full test procedure is intended for determining the experimental standard deviation for a single position and height measurement.

Further, this procedure may be used to determine:

— the measure of the precision of equipment under given conditions (including typical short and long term influences);

— the measure of the precision of equipment used in different periods of time or under different conditions (multiple samples);

— the measure of the capability of comparison between different precision of equipment achievable under similar conditions.

Statistical tests shall be applied to determine whether the sample from the experiment belongs to the same population as the one giving the theoretical standard deviation and to determine whether two samples from different experiments belong to the same population.

5 Simplified test procedure

5.1 Measurements

For the simple test procedure, one series of measurements shall be taken, in which the observer shall obtain five sets of measurements at two rover points. The sequence of the measurements is shown in Table 1 in which the column labelled “Seq. No.” explicitly indicates the sequence of the measurement.

Table 1 — Sequence of the measurements for one series

Seq. No.	Series	Set	Rover point	Measurement		
	i	j	k	x	y	h
1	1	1	1	$x_{1,1,1}$	$y_{1,1,1}$	$h_{1,1,1}$
2	1	1	2	$x_{1,1,2}$	$y_{1,1,2}$	$h_{1,1,2}$
3	1	2	1	$x_{1,2,1}$	$y_{1,2,1}$	$h_{1,2,1}$
4	1	2	2	$x_{1,2,2}$	$y_{1,2,2}$	$h_{1,2,2}$
5	1	3	1	$x_{1,3,1}$	$y_{1,3,1}$	$h_{1,3,1}$
6	1	3	2	$x_{1,3,2}$	$y_{1,3,2}$	$h_{1,3,2}$
7	1	4	1	$x_{1,4,1}$	$y_{1,4,1}$	$h_{1,4,1}$
8	1	4	2	$x_{1,4,2}$	$y_{1,4,2}$	$h_{1,4,2}$
9	1	5	1	$x_{1,5,1}$	$y_{1,5,1}$	$h_{1,5,1}$
10	1	5	2	$x_{1,5,2}$	$y_{1,5,2}$	$h_{1,5,2}$

A specific set of measurements is expressed as $x_{i,j,k}$, $y_{i,j,k}$ and $h_{i,j,k}$ where x, y and h are coordinates of a local coordinate system. The index i stands for the series number, the index j for the set number and the index k for the rover point number. For example $x_{1,3,2}$ is the x component of the third set of measurement at rover point 2 in the first series.

The sequence of measurements should follow Table 1 in the full test procedure (see 6.1).

5.2 Calculation

The individual measurements are compared directly with the nominal values available so as to detect any measurement with gross error.

For each set j (= 1,…,5) in the series i (= 1), calculate the horizontal distance and height difference between two rover points. Subsequently, calculate their deviations from the nominal values.

$$\begin{aligned} D_{i,j} &= \sqrt{(x_{i,j,2} - x_{i,j,1})^2 + (y_{i,j,2} - y_{i,j,1})^2} \\ \Delta h_{i,j} &= h_{i,j,2} - h_{i,j,1} \\ \varepsilon_{D\ i,j} &= D_{i,j} - D^* \qquad\qquad i = 1,\ j = 1,\ldots,5 \\ \varepsilon_{h\ i,j} &= h_{i,j} - h^* \end{aligned} \tag{1}$$

where

$x_{i,j,k}$, $y_{i,j,k}$, $h_{i,j,k}$	are x, y, and h measurements respectively in the set j at rover point k in series i;
$D_{i,j}$, $\Delta h_{i,j}$	are the calculated horizontal distance and height difference respectively in the set j in series i;
D^*, Δh^*	are nominal values of the horizontal distance and height difference respectively;
$\varepsilon_{D\ i,j}$, $\varepsilon_{h\ i,j}$	are deviations of the horizontal distance and height difference respectively.

If any deviation fails to satisfy either of the two conditions in Equation (2) the inclusion of an outlier (or outliers) in the corresponding measurements is suspected, repeat the test procedure.

$$\begin{aligned} \left|\varepsilon_{D\ i,j}\right| &\leqslant 2{,}5 \times \sqrt{2} \times s_{xy} \\ \left|\varepsilon_{h\ i,j}\right| &\leqslant 2{,}5 \times \sqrt{2} \times s_h \end{aligned} \tag{2}$$

Where s_{xy} and s_h are either the pre-determined standard deviation according to the full test procedure or the values specified by the manufacturer.

6 Full test procedure

6.1 Measurements

For the full test procedure, three series of measurements shall be taken. The sequence of the measurements in each series is identical to the case of the simplified test. The start times of consecutive series shall be separated by at least 90 min.

6.2 Calculation

6.2.1 General

Calculation is performed in two steps. In the first step individual measurements are compared directly with the nominal values available in order to detect any measurement with gross error. The statistical values of interest are calculated in the second step. All procedures of the two steps are described in the adjacent clauses.

6.2.2 Preliminary measurement check

The same procedure previously described in the simplified procedure shall be applied to all measurements in all three series.

6.2.3 Calculation of statistical values

Firstly, applying the least squares adjustment on overall measurements in all series, the estimates of x, y and h for each rover point k (= 1, 2) are calculated as

$$\begin{aligned} \bar{x}_k &= \frac{1}{15}\sum_{i=1}^{3}\sum_{j=1}^{5} x_{i,j,k} \\ \bar{y}_k &= \frac{1}{15}\sum_{i=1}^{3}\sum_{j=1}^{5} y_{i,j,k} \qquad k = 1, 2 \\ \bar{h}_k &= \frac{1}{15}\sum_{i=1}^{3}\sum_{j=1}^{5} h_{i,j,k} \end{aligned} \tag{3}$$

Then the residuals of x, y and h for all measurements in three series are calculated as

$$\begin{aligned} r_{x\,i,j,k} &= \bar{x}_k - x_{i,j,k} \\ r_{y\,i,j,k} &= \bar{y}_k - y_{i,j,k} \qquad k = 1, 2, \;\; j = 1,\ldots,5, \;\; i = 1, 2, 3 \\ r_{h\,i,j,k} &= \bar{h}_k - h_{i,j,k} \end{aligned} \tag{4}$$

The above residuals are all squared and summed including measurements for all point index $k = 1$ and $k = 2$ for x, y and h separately as

$$\begin{aligned} \sum r_x^2 &= \sum_{i=1}^{3}\sum_{j=1}^{5}\sum_{k=1}^{2} r_{x\,i,j,k}^2 \\ \sum r_y^2 &= \sum_{i=1}^{3}\sum_{j=1}^{5}\sum_{k=1}^{2} r_{y\,i,j,k}^2 \\ \sum r_h^2 &= \sum_{i=1}^{3}\sum_{j=1}^{5}\sum_{k=1}^{2} r_{h\,i,j,k}^2 \end{aligned} \tag{5}$$

The degrees of freedom for x, y and h are identical.These are calculated as

$$\nu_x = \nu_y = \nu_h = (m \times n - 1) \times p = (3 \times 5 - 1) \times 2 = 28 \tag{6}$$

where

m is the number of series, = 3;

n is the number of sets in a series, = 5;

p is the number of rover points, = 2.

Finally, the standard deviations of a single measurement of x, y and h are calculated as

$$s_x = \sqrt{\frac{\sum r_x^2}{\nu_x}} = \sqrt{\frac{\sum r_x^2}{28}}$$
$$s_y = \sqrt{\frac{\sum r_y^2}{\nu_y}} = \sqrt{\frac{\sum r_y^2}{28}} \quad (7)$$
$$s_h = \sqrt{\frac{\sum r_h^2}{\nu_h}} = \sqrt{\frac{\sum r_h^2}{28}}$$

and they are related to the ISO standard deviations as

$$s_{\text{ISO-GNSS RTK-}xy} = \sqrt{s_x^2 + s_y^2} \quad (8)$$

$$s_{\text{ISO-GNSS RTK-}h} = s_h \quad (9)$$

where

$s_{\text{ISO-GNSS RTK-}xy}$ is the experimental standard deviation of a single position (x, y);

$s_{\text{ISO-GNSS RTK-}h}$ is the experimental standard deviation of a single height (h).

6.3 Statistical tests

6.3.1 General

Statistical tests are practicable for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using the overall standard deviations $s_{\text{ISO-GNSSS RTK-}xy}$, $s_{\text{ISO-GPS RTK-}h}$ obtained from the measurements and their respective degrees of freedom in order to answer the following questions (see Table 2).

a) Is the calculated experimental standard deviation, $s_{\text{ISO-GNSS RTK-}xy}$, of a single position, x, y, smaller than or equal to a corresponding value, σ_{xy}, stated by the manufacturer or another predetermined value, σ_{xy}?

b) Is the calculated experimental standard deviation, $s_{\text{ISO-GNSS RTK-}h}$, of a single height, h, smaller than or equal to a corresponding value, σ_h, stated by the manufacturer or another predetermined value, σ_h?

c) Do two experimental standard deviations, $s_{\text{ISO-GNSS RTK-}xy}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}xy}$, of a single position ($x$, y) as determined from two different samples of measurement belong to the same population, assuming that both samples have the same number of degrees of freedom, $\nu_x + \nu_y$ and $\tilde{\nu}_x + \tilde{\nu}_y$, corresponding to $s_{\text{ISO-GNSS RTK-}xy}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}xy}$ respectively?

d) Do two experimental standard deviations, $s_{\text{ISO-GNSS RTK-}h}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}h}$, of a single height, h, as determined from two different samples of measurement belong to the same population, assuming that both samples have the same number of degrees of freedom, ν_h and $\tilde{\nu}_h$, corresponding to $s_{\text{ISO-GNSS RTK-}h}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}h}$ respectively?

The experimental standard deviations, s and $\tilde{s}$, may be obtained from:

— two samples of measurements by the same equipment;

— two samples of measurements by different equipment.

For the following tests a confidence level of $1-\alpha = 0{,}95$ and the degrees of freedom of $\nu_x + \nu_y = 56$ or $\nu_h = 28$ are assumed according to the design of measurements.

Table 2 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s_{\text{ISO-GNSS RTK-}xy} \leqslant \sigma_{xy}$	$s_{\text{ISO-GNSS RTK-}xy} > \sigma_{xy}$
b)	$s_{\text{ISO-GNSS RTK-}h} \leqslant \sigma_h$	$s_{\text{ISO-GNSS RTK-}h} > \sigma_h$
c)	$\sigma_{xy} = \tilde{\sigma}_{xy}$	$\sigma_{xy} \neq \tilde{\sigma}_{xy}$
d)	$\sigma_h = \tilde{\sigma}_h$	$\sigma_h \neq \tilde{\sigma}_h$

6.3.2 Question a)

The null hypothesis stating that the experimental standard deviation $s_{\text{ISO-GNSS RTK-}xy}$, of a single position, x, y, is smaller than or equal to a theoretical or a predetermined value, σ_{xy}, is not rejected if the following condition is fulfilled:

$$s_{\text{ISO-GNSS RTK-}xy} \leqslant \sigma_{xy} \times \sqrt{\frac{\chi^2_{0,95}(\nu_x + \nu_y)}{\nu_x + \nu_y}} \tag{10}$$

$$s_{\text{ISO-GNSS RTK-}xy} \leqslant \sigma_{xy} \times \sqrt{\frac{\chi^2_{0,95}(56)}{56}} \tag{11}$$

$$\chi^2_{0,95}(56) = 74{,}47 \tag{12}$$

$$s_{\text{ISO-GNSS RTK-}xy} \leqslant \sigma_{xy} \times \sqrt{\frac{74{,}47}{56}} = \sigma_{xy} \times 1{,}15 \tag{13}$$

Otherwise, the null hypothesis is rejected.

6.3.3 Question b)

The null hypothesis stating that the experimental standard deviation, $s_{\text{ISO-GNSS RTK-}h}$, of a single height, h, is smaller than or equal to a theoretical or a predetermined value, σ_h, is not rejected if the following condition is fulfilled:

$$s_{\text{ISO-GNSS RTK-}h} \leqslant \sigma_h \times \sqrt{\frac{\chi^2_{0,95}(\nu_h)}{\nu_h}} \tag{14}$$

$$s_{\text{ISO-GNSS RTK-}h} \leqslant \sigma_h \times \sqrt{\frac{\chi^2_{0,95}(28)}{28}} \tag{15}$$

$$\chi^2_{0,95}(28) = 41{,}34 \tag{16}$$

$$s_{\text{ISO-GNSS RTK-}h} \leqslant \sigma_h \times \sqrt{\frac{41{,}34}{28}} = \sigma_h \times 1{,}22 \tag{17}$$

Otherwise, the null hypothesis is rejected.

6.3.4 Question c)

In the case of two different samples, the test indicates whether the experimental standard deviations, $s_{\text{ISO-GNSS RTK-}xy}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}xy}$, of a single position (x, h) belong to the same population. The corresponding null hypothesis, $\sigma_{xy} = \tilde{\sigma}_{xy}$, is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\tilde{\nu}_x + \tilde{\nu}_y, \nu_x + \nu_y)} \leqslant \frac{s^2_{\text{ISO-GNSS RTK-}xy}}{\tilde{s}^2_{\text{ISO-GNSS RTK-}xy}} \leqslant F_{1-\alpha/2}(\nu_x + \nu_y, \tilde{\nu}_x + \tilde{\nu}_y) \tag{18}$$

$$\frac{1}{F_{0,975}(56,56)} \leqslant \frac{s^2_{\text{ISO-GNSS RTK-}xy}}{\tilde{s}^2_{\text{ISO-GNSS RTK-}xy}} \leqslant F_{0,975}(56, 56) \tag{19}$$

$$F_{0,975}(56, 56) = 1{,}70 \tag{20}$$

$$0{,}59 \leqslant \frac{s^2_{\text{ISO-GNSS RTK-}xy}}{\tilde{s}^2_{\text{ISO-GNSS RTK-}xy}} \leqslant 1{,}70 \tag{21}$$

Otherwise, the null hypothesis is rejected.

6.3.5 Question d)

The hypothesis that the two experimental standard deviations, $s_{\text{ISO-GNSS RTK-}h}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}h}$, , of a single height, h, belong to the same population is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\tilde{\nu}_h, \nu_h)} \leqslant \frac{s^2_{\text{ISO-GNSS RTK-}h}}{\tilde{s}^2_{\text{ISO-GNSS RTK-}h}} \leqslant F_{1-\alpha/2}(\nu_h, \tilde{\nu}_h) \tag{22}$$

$$\frac{1}{F_{0,975}(28,28)} \leqslant \frac{s^2_{\text{ISO-GNSS RTK-}h}}{\tilde{s}^2_{\text{ISO-GNSS RTK-}h}} \leqslant F_{0,975}(28,28) \tag{23}$$

$$F_{0,975}(28,28) = 2{,}13 \tag{24}$$

$$0{,}47 \leqslant \frac{s^2_{\text{ISO-GNSS RTK-}h}}{\tilde{s}^2_{\text{ISO-GNSS RTK-}h}} \leqslant 2{,}13 \tag{25}$$

Otherwise, the null hypothesis is rejected

Annex A
(informative)

Example of the simplified test procedure

A.1 Measurements

— Observer: Tokio

— Weather: Fine, + 5 °C

— Instruments type and number: AAA 01234

— Antenna type and number or built in: BBB 05678

— Date: 2006-01-21

— Base line (nominal value) horizontal distance D^* :19,996 m, height difference Δh^*: 0,038 m

— Pre-defined standard deviation $s_{xy} = 15$ mm; $s_h = 25$ mm

A.2 Calculation

According to Equation (1), the calculated data are shown in the Table A.1

Table A.1 — Measurements and deviations

Seq. No.	Set j	Rover point k	Measurements m x	y	h	Horizontal distance D_j m	Height difference Δh_j m	Deviations mm $\varepsilon_{D,i,j}$	$\varepsilon_{h,i,j}$
1	1	1	– 67 637,433	– 63 945,554	320,732	—	—	—	—
2	1	2	– 67 654,082	– 63 934,442	320,781	20,017	0,049	21	11
3	2	1	– 67 637,448	– 63 945,550	320,732	—	—	—	—
4	2	2	– 67 654,084	– 63 934,451	320,774	19,999	0,042	3	4
5	3	1	– 67 637,450	– 63 945,550	320,745	—	—	—	—
6	3	2	– 67 654,083	– 63 934,454	320,793	19,994	0,048	– 2	10
7	4	1	– 67 637,453	– 63 945,541	320,731	—	—	—	—
8	4	2	– 67 654,077	– 63934,447	320,783	19,986	0,052	– 10	14
9	5	1	– 67 637,450	– 63 945,555	320,740	—	—	—	—
10	5	2	– 67 654,083	– 63 934,452	320,778	19,998	0,038	2	0
Limit of each deviation mm			—	—	—	—	—	± 53	± 88

All deviations satisfy the condition of Equation (2). No outlier is suspected.

Annex B
(informative)

Example of the full test procedure

B.1 Measurements

- Observer: Bonn
- Weather: Fine + 5 °C
- Instruments type and number: BBB 01234
- Antenna type and number or built in: CCC 05678
- Date: 2006-09-22
- Base line (nominal value) horizontal distance D^* :19,994 m, height difference Δh^* : 0,028 m
- Pre-defined standard deviation s_{xy} = 15 mm; s_h = 25 mm

B.2 Calculation

B.2.1 Preliminary check

According to the Equation (1), the calculated data are shown on the Table B.1

Table B.1 — Measurements and deviations

Seq. No.	Series	Set	Rover point	Measurement m			Horizontal distance D_j	Height difference Δh_j	Deviations mm	
	i	j	k	x	y	h	m	m	$\varepsilon_{D,i,j}$	$\varepsilon_{h,i,j}$
1	1	1	1	– 67 635,470	– 63 943,197	320,792	—	—	—	—
2	1	1	2	– 67 652,389	– 63 932,527	320,799	20,003	0,007	9	– 21
3	1	2	1	– 67 635,479	– 63 943,188	320,788	—	—	—	—
4	1	2	2	– 67 652,376	– 63 932,525	320,824	19,980	0,036	– 14	8
5	1	3	1	– 67 635,480	– 63 943,189	320,789	—	—	—	—
6	1	3	2	– 67 652,387	– 63 932,529	320,810	19,987	0,021	– 7	– 7
7	1	4	1	– 67 635,476	– 63 943,192	320,793	—	—	—	—
8	1	4	2	– 67 652,393	– 63 932,530	320,808	19,997	0,015	3	– 13
9	1	5	1	– 67 635,481	– 63 943,192	320,794	—	—	—	—
10	1	5	2	– 67 652,390	– 63 932,522	320,803	19,994	0,009	0	– 19
11	2	1	1	– 67 635,478	– 63 943,191	320,800	—	—	—	—
12	2	1	2	– 67 652,399	– 63 932,535	320,823	19,997	0,023	3	– 5
13	2	2	1	– 67 635,479	– 63 943,193	320,798	—	—	—	—
14	2	2	2	– 67 652,392	– 63 932,528	320,828	19,995	0,030	1	2
15	2	3	1	– 67 635,477	– 63 943,194	320,780	—	—	—	—
16	2	3	2	– 67 652,396	– 63 932,530	320,797	19,999	0,017	5	– 11
17	2	4	1	– 67 635,475	– 63 943,191	320,786	—	—	—	—
18	2	4	2	– 67 652,395	– 63 932,532	320,812	19,998	0,026	4	– 2
19	2	5	1	– 67 635,476	– 63 943,191	320,784	—	—	—	—
20	2	5	2	– 67 652,391	– 63 932,534	320,812	19,992	0,028	– 2	0
21	3	1	1	– 67 635,479	– 63 943,194	320,798	—	—	—	—
22	3	1	2	– 67 652,391	– 63 932,529	320,826	19,994	0,028	0	0
23	3	2	1	– 67 635,478	– 63 943,195	320,805	—	—	—	—
24	3	2	2	– 67 652,398	– 63 932,532	320,823	20,000	0,018	6	– 10
25	3	3	1	– 67 635,485	– 63 943,199	320,799	—	—	—	—
26	3	3	2	– 67 652,400	– 63 932,534	320,813	19,996	0,014	2	– 14
27	3	4	1	– 67 635,474	– 63 943,195	320,804	—	—	—	—
28	3	4	2	– 67 652,394	– 63 932,532	320,831	20,000	0,027	6	– 1
29	3	5	1	– 67 635,483	– 63 943,200	320,793	—	—	—	—
30	3	5	2	– 67 652,398	– 63 932,537	320,833	19,995	0,040	1	12
Limit of each deviation mm				—	—	—	—	—	± 53	± 88

All deviations satisfy the conditions of Equation (2). No outlier is suspected.

B.2.2 Calculation of statistical values

Table B.2 — Measurements, residuals and standard deviation

Seq. No.	Series	Set	Rover point	Measurement m			Residual mm			Squared residual mm²		
	i	j	k	x	y	h	r_x	r_y	r_h	r_x^2	r_y^2	r_h^2
1	1	1	1	– 67 635,470	– 63 943,197	320,792	8	– 4	– 2	64	16	4
2	1	1	2	– 67 652,389	– 63 932,527	320,799	4	3	– 17	16	9	289
3	1	2	1	– 67 635,479	– 63 943,188	320,788	– 1	5	– 6	1	25	36
4	1	2	2	– 67 652,376	– 63 932,525	320,824	17	5	8	289	25	64
5	1	3	1	– 67 635,480	– 63 943,189	320,789	– 2	4	– 5	4	16	25
6	1	3	2	– 67 652,387	– 63 932,529	320,810	6	1	– 6	36	1	36
7	1	4	1	– 67 635,476	– 63 943,192	320,793	2	1	– 1	4	1	1
8	1	4	2	– 67 652,393	– 63 932,530	320,808	0	0	– 8	0	0	64
9	1	5	1	– 67 635,481	– 63 943,192	320,794	– 3	1	0	9	1	0
10	1	5	2	– 67 652,390	– 63 932,522	320,803	3	8	– 13	9	64	169
11	2	1	1	– 67 635,478	– 63 943,191	320,800	0	2	6	0	4	36
12	2	1	2	– 67 652,399	– 63 932,535	320,823	– 6	– 4	7	36	16	49
13	2	2	1	– 67 635,479	– 63 943,193	320,798	– 1	0	4	1	0	16
14	2	2	2	– 67 652,392	– 63 932,528	320,828	1	2	12	1	4	144
15	2	3	1	– 67 635,477	– 63 943,194	320,780	1	– 1	– 14	1	1	196
16	2	3	2	– 67 652,396	– 63 932,530	320,797	– 3	0	– 19	9	0	361
17	2	4	1	– 67 635,475	– 63 943,191	320,786	3	2	– 8	9	4	64
18	2	4	2	– 67 652,395	– 63 932,532	320,812	– 2	– 2	– 4	4	4	16
19	2	5	1	– 67 635,476	– 63 943,191	320,784	– 2	– 2	–10	4	4	100
20	2	5	2	– 67 652,391	– 63 932,534	320,812	2	– 4	– 4	4	16	16
21	3	1	1	– 67 635,479	– 63 943,194	320,798	– 1	– 1	4	1	1	16
22	3	1	2	– 67 652,391	– 63 932,529	320,826	2	1	10	4	1	100
23	3	2	1	– 67 635,478	– 63 943,195	320,805	0	– 2	11	0	4	121
24	3	2	2	– 67 652,398	– 63 932,532	320,823	– 5	– 2	7	25	4	49
25	3	3	1	– 67 635,485	– 63 943,199	320,799	– 7	– 6	5	49	36	25
26	3	3	2	– 67 652,400	– 63 932,534	320,813	– 7	– 4	– 3	49	16	9
27	3	4	1	– 67 635,474	– 63 943,195	320,804	4	– 2	10	16	4	100
28	3	4	2	– 67 652,394	– 63 932,532	320,831	– 1	– 2	15	1	4	225
29	3	5	1	– 67 635,483	– 63 943,200	320,793	– 5	– 7	– 1	25	49	1
30	3	5	2	– 67 652,398	– 63 932,537	320,833	– 5	– 7	17	25	49	289
Average over the series			1	– 67 635,478	– 63 943,193	320,794	—	—	—	—	—	—
			2	– 67 652,393	– 63 932,530	320,816						
Summation of the squared residual				—	—	—	—	—	—	696	379	2621
Experimental standard deviation, s				$s_x = 4{,}99$	$s_y = 3{,}68$	$s_h = 9{,}68$	—	—	—	—	—	—

According to Equation (6), the degrees of freedom for x, y and h are identical and they are calculated as

$$\nu_x = \nu_y = \nu_h = (m \times n - 1) \times p = (3 \times 5 - 1) \times 2 = 28$$

According to Equation (7), the standard deviations of a single measurement of x, y and h are calculated as

$$s_x = \sqrt{\frac{\sum r_x^2}{\nu_x}} = \sqrt{\frac{\sum r_x^2}{28}} = \sqrt{\frac{696}{28}} = 4{,}99 \text{ mm}$$

$$s_y = \sqrt{\frac{\sum r_y^2}{\nu_y}} = \sqrt{\frac{\sum r_y^2}{28}} = \sqrt{\frac{379}{28}} = 3{,}68 \text{ mm}$$

$$s_h = \sqrt{\frac{\sum r_h^2}{\nu_h}} = \sqrt{\frac{\sum r_h^2}{28}} = \sqrt{\frac{2621}{28}} = 9{,}68 \text{ mm}$$

According to Equations (8) and (9) related ISO standard deviations are

$$s_{\text{ISO-GNSS RTK-}xy} = \sqrt{\sum s_x^2 + \sum s_y^2} = \sqrt{4{,}99^2 + 3{,}68^2} = 6{,}20 \text{ mm}$$

$$s_{\text{ISO-GNSS RTK-}h} = s_h = 9{,}68 \text{ mm}$$

B.3 Statistical tests

B.3.1 Statistical test according to question a)

$$s_{\text{ISO-GNSS RTK-}xy} = 6{,}20 \text{ mm}; \sigma_{xy} = 15{,}00 \text{ mm}; \nu = \nu_x + \nu_y = 56$$

$$6{,}20 \leqslant 15{,}00 \times 1{,}15$$

$$6{,}20 \leqslant 17{,}2$$

The test condition is fulfilled, the calculated experimental standard deviation, $s_{\text{ISO-GNSS RTK-}xy}$, of a single position ($x$, y) is smaller than or equal to a corresponding value, σ_{xy}, stated by the manufacturer or another predetermined value, σ_{xy}.

B.3.2 Statistical test according to question b)

$$s_{\text{ISO-GNSS RTK-}h} = s_h = 9{,}68 \text{ mm}; \quad \sigma_h = 25{,}00 \text{ mm}; \quad \nu = 28$$

$$9{,}68 \leqslant 25{,}00 \times 1{,}22$$

$$9{,}68 \leqslant 30{,}5$$

The test condition is fulfilled, the calculated experimental standard deviation, $s_{\text{ISO-GNSS RTK-}h}$, of a single height, h, is smaller than or equal to a corresponding value, σ_h, stated by the manufacture or another predetermined value, σ_h.

B.3.3 Statistical test according to question c)

$$s_{\text{ISO-GNSS RTK-}xy} = 6{,}20 \text{ mm}; \tilde{s}_{\text{ISO-GNSS RTK-}xy} = 6{,}00 \text{ mm}; \nu = \nu_x + \nu_y = 56$$

$$0{,}59 \leqslant \frac{38{,}44}{36{,}00} \leqslant 1{,}70$$

$$0{,}59 \leqslant 1{,}07 \leqslant 1{,}70$$

The test condition is fulfilled, the null hypothesis stating that the standard deviations, $s_{\text{ISO-GNSS RTK-}xy}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}xy}$, belong to the same population is not rejected at the confidence level of 95 %.

B.3.4 Statistical test according to question d)

$s_{\text{ISO-GNSS RTK-}h} = 9{,}68$ mm; $\tilde{s}_{\text{ISO-GNSS RTK-}h} = 10{,}00$ mm; $\nu_h = 28$

$$0{,}47 \leqslant \frac{93{,}70}{100{,}00} \leqslant 2{,}13$$

$$0{,}47 \leqslant 0{,}94 \leqslant 2{,}13$$

The test condition is fulfilled, the null hypothesis stating that the standard deviations, $s_{\text{ISO-GNSS RTK-}h}$ and $\tilde{s}_{\text{ISO-GNSS RTK-}h}$, belong to the same population is not rejected at the confidence level of 95 %.

It can be concluded that the 3-dimentional position measurement obtained by different samples and separated into position, xy, and in height, h, is not influenced by any biases.